

3. sp.:

$$\int \sqrt{8t^2 + 1} dt = \int \sqrt{(\sqrt{8}t)^2 + 1} dt = \left. \begin{array}{l} t = \frac{1}{\sqrt{8}} \operatorname{sh} x \quad \operatorname{ch}^2 x - \operatorname{sh}^2 x = 1 \\ dt = \frac{1}{\sqrt{8}} \operatorname{ch} x \\ x = \operatorname{arsinh}(\sqrt{8} \cdot t) \end{array} \right| =$$

$$\frac{1}{\sqrt{8}} \int \sqrt{\operatorname{sh}^2 x + 1} \operatorname{ch} x dx = \frac{1}{\sqrt{8}} \int \operatorname{ch} x \cdot \operatorname{ch} x dx$$

NR:

$$\int \frac{du}{u} \cdot \frac{1}{v} = \operatorname{sh} x \operatorname{ch} x - \int \operatorname{sh}^2 x dx = \operatorname{sh} x \cdot \operatorname{ch} x - \int (\operatorname{ch}^2 x - 1) dx$$

$$= \operatorname{sh} x \cdot \operatorname{ch} x - \int \operatorname{ch}^2 x + x + C$$

$$\Rightarrow 2 \int \operatorname{ch}^2 x dx = \operatorname{sh} x \cdot \operatorname{ch} x + x + C$$

$$\Rightarrow \int \operatorname{ch}^2 x dx = \frac{1}{2} \operatorname{sh} x \cdot \operatorname{ch} x + \frac{x}{2} + C$$

$$\Rightarrow \frac{1}{\sqrt{8}} \int \operatorname{ch}^2 x dx = \frac{1}{2 \cdot \sqrt{8}} \operatorname{sh} x \cdot \operatorname{ch} x + \frac{x}{2 \sqrt{8}} + C$$

$$= \frac{1}{2 \sqrt{8}} \operatorname{sh} x \sqrt{1 + \operatorname{sh}^2 x} + \frac{x}{2 \sqrt{8}} + C$$

$$= \frac{1}{2 \sqrt{8}} \sqrt{8} \cdot t \sqrt{1 + 8t^2} + \frac{\operatorname{arsinh}(\sqrt{8} \cdot t)}{2 \sqrt{8}} + C$$

$x = \operatorname{arsinh}(\sqrt{8}t)$

$\operatorname{arsinh} x = \ln(x + \sqrt{x^2 + 1})$

$$\Rightarrow \int_0^1 \sqrt{8t^2 + 1} dt = \left[ \frac{t}{2} \cdot \sqrt{1 + 8t^2} + \frac{\operatorname{arsinh}(\sqrt{8} \cdot t)}{2 \sqrt{8}} \right]_0^1 =$$

$$\ln \frac{1}{2} \cdot 3 + \frac{\operatorname{arcsinh}(\sqrt{8})}{2\sqrt{8}} \approx \underline{\underline{1.811}}$$