Lösung zu Aufg. 16

<u>geg.:</u> $|\vec{a}| = 3$, $|\vec{b}| = 5$, $|\vec{c}| = 2$, $\sphericalangle(\vec{a},\vec{b}) = 120°$

$\sphericalangle(\vec{b},\vec{c}) = 60°$, $\sphericalangle(\vec{c},\vec{a}) = 120°$

<u>zu a)</u>: Flächendiagonale von $\vec{a}, \vec{b}$: $\vec{d} = \vec{a} + \vec{b}$

$$|\vec{d}| = |\vec{a}+\vec{b}| = \sqrt{(\vec{a}+\vec{b})^2} = \sqrt{\vec{a}^2 + 2\vec{a}\cdot\vec{b} + \vec{b}^2}$$

$$= \sqrt{|\vec{a}|^2 + 2|\vec{a}|\cdot|\vec{b}|\cdot\cos 120° + |\vec{b}|^2} = \sqrt{9 + 2\cdot 3\cdot 5\left(-\tfrac{1}{2}\right) + 25}$$

$$= \sqrt{19}$$

<u>zu b)</u>: $|\vec{d}| = |\vec{a}+\vec{b}+\vec{c}| = \sqrt{(\vec{a}+\vec{b}+\vec{c})(\vec{a}+\vec{b}+\vec{c})}$

$$= \sqrt{\vec{a}^2 + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{a} + \vec{b}^2 + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} + \vec{c}\cdot\vec{b} + \vec{c}^2}$$

$$= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2\vec{a}\cdot\vec{c} + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c}}$$

$$= \sqrt{|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 2|\vec{a}|\cdot|\vec{c}|\cdot\cos 120° + 2|\vec{a}|\,|\vec{b}|\cos 120° + 2|\vec{b}|\cdot|\vec{c}|\cdot\cos 60°}$$

$$= \sqrt{9 + 25 + 4 + 12\left(-\tfrac{1}{2}\right) + 30\left(-\tfrac{1}{2}\right) + 20\left(\tfrac{1}{2}\right)} = \sqrt{27} = 3\sqrt{3}$$

<u>zu c)</u>: $\alpha = \sphericalangle(\vec{d},\vec{a})$, $\beta = \sphericalangle(\vec{d},\vec{b})$, $\gamma = \sphericalangle(\vec{d},\vec{c})$

$$\cos\alpha = \frac{\vec{d}\cdot\vec{a}}{|\vec{d}|\cdot|\vec{a}|}, \quad \vec{d}\cdot\vec{a} = (\vec{a}+\vec{b}+\vec{c})\cdot\vec{a}$$

$$= |\vec{a}|^2 + \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$$

$$= |\vec{a}|^2 + |\vec{a}|\cdot|\vec{b}|\cdot\cos 120°$$

$$+ |\vec{a}|\cdot|\vec{c}|\cdot\cos 120°$$

$$\Rightarrow \quad \vec{d}\cdot\vec{a} = 9 + 3\cdot 5\left(-\tfrac{1}{2}\right) + 3\cdot 2\cdot\left(-\tfrac{1}{2}\right) = -\tfrac{3}{2}$$

$$\Rightarrow \quad \cos\alpha = -\frac{3}{2\cdot 3\cdot\sqrt{3}\cdot 3} = -\frac{1}{6\sqrt{3}} \Rightarrow \alpha = 95{,}52^\circ$$

Berechne entsprechend $\cos\beta$, $\cos\gamma$:

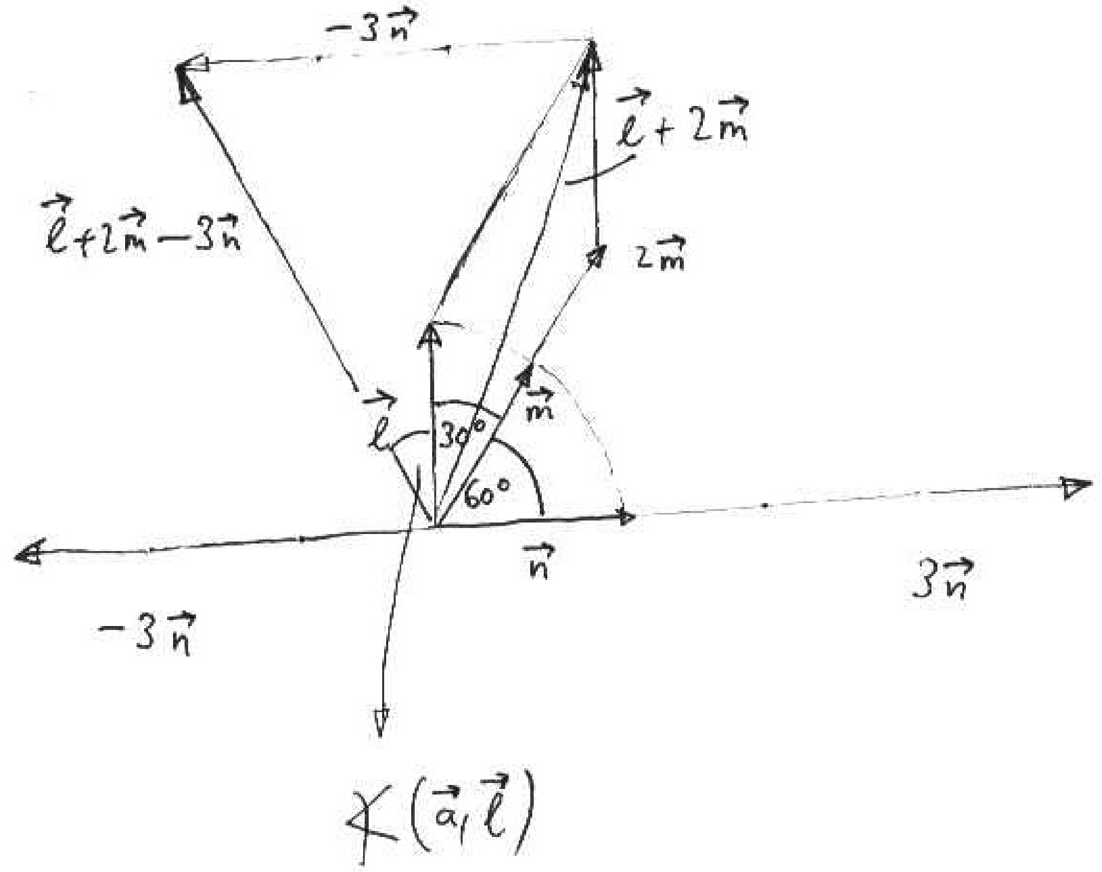
$$\cos\beta = \frac{\vec{d}\cdot\vec{b}}{|\vec{d}|\cdot|\vec{b}|} \Rightarrow \beta = 30^\circ, \quad \cos\gamma = \frac{\vec{d}\cdot\vec{c}}{|\vec{d}||\vec{c}|} \Rightarrow \gamma = 54{,}74^\circ$$

zu d): $d_a = |\vec{d}|\cdot\cos\alpha = 3\cdot\sqrt{3}\left(-\frac{1}{6\sqrt{3}}\right) = -\frac{1}{2}$

$d_b = |\vec{d}|\cdot\cos\beta = \frac{9}{2}$, $\quad d_c = |\vec{d}|\cos\gamma = 3$

$\boxed{\text{Lösung zu Aufg. 19}}$ $\quad \vec{a} = \vec{l} + 2\vec{m} - 3\vec{n}$

zu a): hier gilt: $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$



zu b):

$$\cos \sphericalangle(\vec{a},\vec{\ell}) = \frac{\vec{a}\cdot\vec{\ell}}{|\vec{a}||\vec{\ell}|} = \frac{(\vec{\ell}+2\vec{m}-3\vec{n})\vec{\ell}}{|\vec{a}||\vec{\ell}|}$$

$$|\vec{a}| = \sqrt{(\vec{\ell}+2\vec{m}-3\vec{n})(\vec{\ell}+2\vec{m}-3\vec{n})}$$

$$= \sqrt{|\vec{\ell}|^2 + 2\,\vec{\ell}\cdot\vec{m} - 3\,\vec{\ell}\cdot\vec{n} + 2\,\vec{m}\cdot\vec{\ell} + 4|\vec{m}|^2 - 6\,\vec{m}\cdot\vec{n} - 3\,\vec{n}\cdot\vec{\ell} - 6\,\vec{m}\cdot\vec{n} + 9|\vec{n}|^2} =$$

$$= \sqrt{|\vec{\ell}|^2 + 4\vec{m}\cdot\vec{\ell} - 6\,\vec{n}\cdot\vec{\ell} - 12\,\vec{m}\cdot\vec{n} + 4|\vec{m}|^2 + 9|\vec{n}|^2} =$$

$$= \sqrt{|\vec{\ell}|^2 + 4|\vec{m}||\vec{\ell}|\cdot\cos 30^\circ - 6|\vec{n}||\vec{\ell}|\cdot\underbrace{\cos 90^\circ}_{=0} - 12|\vec{m}||\vec{n}|\cdot\cos 60^\circ + 4|\vec{m}|^2 + 9|\vec{n}|^2} =$$

$$= \sqrt{1 + 4\cdot\tfrac{1}{2}\sqrt{3} - 12\cdot\tfrac{1}{2} + 4\cdot 9} = \sqrt{8 + 2\sqrt{3}}$$

$$(\vec{\ell}+2\vec{m}-3\vec{n})\cdot\vec{\ell} = \vec{a}\cdot\vec{\ell} = |\vec{\ell}|^2 + 2\vec{m}\cdot\vec{\ell} - 3\vec{n}\cdot\vec{\ell} =$$

$$= |\vec{\ell}|^2 + 2|\vec{m}||\vec{\ell}|\cdot\cos 30^\circ - 3|\vec{n}||\vec{\ell}|\cdot\underbrace{\cos 90^\circ}_{=0} =$$

$$= 1 + 2\cdot\tfrac{1}{2}\sqrt{3} = 1 + \sqrt{3}$$

$$\Rightarrow \cos \sphericalangle(\vec{a},\vec{\ell}) = \frac{1+\sqrt{3}}{\sqrt{8+2\sqrt{3}}}$$

$$\Rightarrow \sphericalangle(\vec{a},\vec{\ell}) = \arccos\left(\frac{1+\sqrt{3}}{\sqrt{8+2\sqrt{3}}}\right) = \underline{\underline{32{,}2^\circ}}$$

$\boxed{A31}$ $\vec{F}_1 = 100\vec{i}$, $\vec{F}_2 = 150\vec{j}$, $F_3 = -120\vec{k}$

a) $\Rightarrow \vec{F}_R = \sum_{i=1}^{3} \vec{F}_i = 100\vec{i} + 150\vec{j} - 120\vec{k}$

$$\Rightarrow F_R = \begin{pmatrix} 100 \\ 150 \\ -120 \end{pmatrix} N$$

$P_1(0|0{,}4|0{,}2)$, $P_2(0{,}2|0{,}4|0)$, $P_3(0{,}2|0{,}4|0{,}2)$ (in m!)

$r_i = \vec{OP_i}$ mit $\vec{r}_1 = \vec{OP_1} = \begin{pmatrix} 0 \\ 0{,}4 \\ 0{,}2 \end{pmatrix}$, $\vec{r}_2 = \vec{OP_2} = \begin{pmatrix} 0{,}2 \\ 0{,}4 \\ 0 \end{pmatrix}$, $\vec{r}_3 = \vec{OP_3} = \begin{pmatrix} 0{,}2 \\ 0{,}4 \\ 0{,}2 \end{pmatrix}$

$$M_0 = \vec{r}_1 \times \vec{F}_1 + \vec{r}_2 \times \vec{F}_2 + \vec{r}_3 \times \vec{F}_3 = \sum_{i=1}^{3} (r_i \times F_i)$$

$\underline{\vec{r}_1 \times \vec{F}_1}$: $\begin{array}{cc} \cancel{0} & \cancel{100} \\ 0{,}4 & 0 \\ 0{,}2 & 0 \\ 0 & 100 \\ 0{,}4 & 0 \\ \cancel{0{,}2} & \cancel{0} \end{array}$ $\begin{pmatrix} 0 \\ 20 \\ -40 \end{pmatrix}$

$\underline{\vec{r}_2 \times \vec{F}_2}$: $\begin{array}{cc} \cancel{0{,}2} & \cancel{0} \\ 0{,}4 & 150 \\ 0 & 0 \\ 0{,}2 & 0 \\ 0{,}4 & 150 \\ \cancel{0} & \cancel{0} \end{array}$ $\begin{pmatrix} 0 \\ 0 \\ 30 \end{pmatrix}$

$\vec{r}_3 \times \vec{F}_3$: $\begin{array}{cc} \cancel{0{,}2} & \cancel{0} \\ 0{,}4 & 0 \\ 0{,}2 & -120 \\ 0{,}2 & 0 \\ 0{,}4 & 0 \\ \cancel{0{,}2} & \cancel{-120} \end{array}$ $\begin{pmatrix} -48 \\ 24 \\ 0 \end{pmatrix}$

$$\Rightarrow M_0 = \begin{pmatrix} 0 \\ 20 \\ -40 \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 30 \end{pmatrix} + \begin{pmatrix} -48 \\ 24 \\ 0 \end{pmatrix}$$

$$\Rightarrow \boxed{M_0 = \begin{pmatrix} -48 \\ 44 \\ -10 \end{pmatrix} Nm}$$

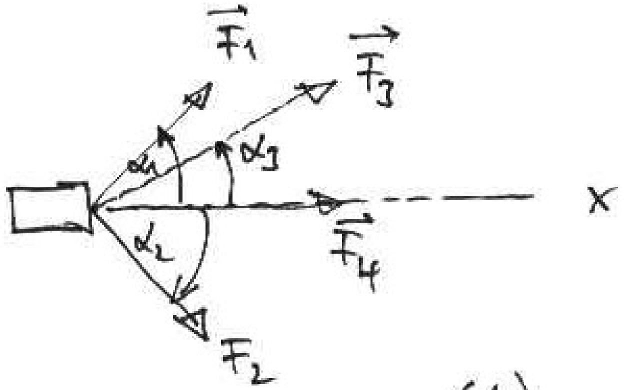
b)

$$|\vec{F}_R| = \sqrt{100^2 + 150^2 + 120^2} = 216{,}56\ N$$

$\cos\varphi_x = \frac{100}{|\vec{F}_R|} \Rightarrow \boxed{\varphi_x = 62{,}5^\circ}$; $\cos\varphi_y = \frac{150}{|\vec{F}_R|} \Rightarrow \boxed{\varphi_y = 46{,}16^\circ}$

$$\cos\varphi_z = \frac{-120}{|\vec{F}_R|} = 123{,}65^\circ$$

B6



$$\vec{F_1} = 700 \begin{pmatrix} \cos 60^\circ \\ \sin 60^\circ \end{pmatrix} N = 700 \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} N$$

$$\vec{F_2} = 600 \begin{pmatrix} \cos(-45^\circ) \\ \sin(-45^\circ) \end{pmatrix} N = 600 \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix} N$$

$$\vec{F_3} = |\vec{F_3}| \begin{pmatrix} \cos \alpha_3 \\ \sin \alpha_3 \end{pmatrix} N, \quad \vec{F_4} = 1000 \begin{pmatrix} \cos 0 \\ \sin 0 \end{pmatrix} N = 1000 \begin{pmatrix} 1 \\ 0 \end{pmatrix} N$$

Es gilt: $\vec{F_1} + \vec{F_2} + \vec{F_3} - \vec{F_4} = \vec{0}$

$$\Rightarrow 700 \begin{pmatrix} \frac{1}{2} \\ \frac{\sqrt{3}}{2} \end{pmatrix} N + 600 \begin{pmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{pmatrix} N + |\vec{F_3}| \begin{pmatrix} \cos \alpha_3 \\ \sin \alpha_3 \end{pmatrix} N - 1000 \begin{pmatrix} 1 \\ 0 \end{pmatrix} N = \begin{pmatrix} 0 \\ 0 \end{pmatrix} N$$

$$\Rightarrow |\vec{F_3}| \begin{pmatrix} \cos \alpha_3 \\ \sin \alpha_3 \end{pmatrix} = \begin{pmatrix} 225{,}74 \\ -181{,}95 \end{pmatrix} N$$

$$\Rightarrow \vec{F_3} = \sqrt{225{,}74^2 + 181{,}95^2}\, N = \boxed{289{,}54\ N}$$

$$\boxed{\alpha = -\arccos\left(\frac{225{,}74}{289{,}54}\right) = -38{,}57^\circ}$$