A1 $\vec{a}, \vec{b}, \vec{c}$ l. a. $\Leftrightarrow$ $\vec{a}, \vec{b}, \vec{c}$ komplanar

a) 5

$\Leftrightarrow [\vec{a}\ \vec{b}\ \vec{c}] = 0$ ✓

$$\Rightarrow \begin{vmatrix} 1 & 1 & 5 \\ -1 & t & 0 \\ 1 & 1 & t+3 \end{vmatrix} = 0 \quad |(-1) \quad \tfrac{1}{2} \quad \Leftrightarrow \begin{vmatrix} 1 & 1 & 5 \\ 0 & t+1 & 5 \\ 0 & 0 & t-2 \end{vmatrix} = 0 \quad \tfrac{1}{2}$$

$$\Leftrightarrow (t+1)(t-2) = 0 \Leftrightarrow \boxed{t=-1} \vee \boxed{t=2}$$ ✓ ✓ ✓

b) 7 $\vec{a}, \vec{b}$ spannen Q. auf falls $|\vec{a}| = |\vec{b}| \wedge \vec{a} \perp \vec{b}$ ½ ½

$\Rightarrow (1)\ |\vec{a}| = \sqrt{4p^2+p^2+4p^2} = \sqrt{9p^2}$ ½

$(2)\ |\vec{b}| = \sqrt{4+16+q^2} = \sqrt{20+q^2}$ ½

$\vec{a} \perp \vec{b} \Leftrightarrow \vec{a}\cdot\vec{b} = 0 \Leftrightarrow 4p+4p+2pq = 0$ ✓

$\Leftrightarrow 8p+2pq = 0 \Leftrightarrow 4p+pq = 0 \Leftrightarrow p(4+q) = 0$

$\Leftrightarrow p = 0 \vee q = -4$ ✓ Da $p \neq 0 \Rightarrow (3)\ \boxed{q=-4}$

(3) in (2): $|\vec{b}| = \sqrt{20+16} = 6$ ✓

$|\vec{b}| = 6 \overset{!}{=} \sqrt{9p^2} \Rightarrow 9p^2 = 36 \Rightarrow p = \pm 2$ ✓

$\Rightarrow [p=2 \wedge q=-4] \vee [p=-2 \wedge q=-4]$ ✓

c) 4 $|(\vec{a}+\vec{b})\cdot(\vec{a}-\vec{b})| = \left||\vec{a}|^2 - \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{b} - |\vec{b}|^2\right|$ ✓✓

$= \left||\vec{a}|^2 - |\vec{b}|^2\right| = |4-9| = 5$ ✓ ✓

d) 4

$$(\vec{a}+p\vec{b}) \perp \vec{c} \iff (\vec{a}+p\vec{b})\cdot\vec{c} = 0$$

$$\iff \vec{a}\cdot\vec{c} + p\,\vec{b}\cdot\vec{c} = 0 \quad \tfrac{1}{2} \iff p\,\vec{b}\cdot\vec{c} = -\,\vec{a}\cdot\vec{c} \quad \tfrac{1}{2}$$

$$\iff \boxed{p = -\,\frac{\vec{a}\cdot\vec{c}}{\vec{b}\cdot\vec{c}}}$$

e) 8

$$A_\Delta = \frac{1}{2}\,|\vec{a}\times\vec{b}|$$

$$\vec{a} = \begin{pmatrix}1\\2\\-1\end{pmatrix} \qquad \vec{b} = \begin{pmatrix}k\\0\\1\end{pmatrix}$$

$$\Rightarrow \quad \vec{a}\times\vec{b} = \begin{pmatrix}2\\-k-1\\-2k\end{pmatrix}$$

$$|\vec{a}\times\vec{b}| = \sqrt{4+(1+k)^2+4k^2} = \sqrt{5k^2+2k+5}$$

$$\Rightarrow \quad A_\Delta = \frac{1}{2}\sqrt{5k^2+2k+5} \overset{!}{=} \sqrt{2}$$

$$\Rightarrow \quad \sqrt{5k^2+2k+5} = 2\sqrt{2}$$

$$\Rightarrow \quad 5k^2+2k-3 = 0$$

$$\Rightarrow \quad k_{1/2} = \frac{-2 \pm \overbrace{\sqrt{4+60}}^{8}}{10}$$

$$\Rightarrow \quad \boxed{k_1 = \frac{3}{5}} \;\vee\; \boxed{k_2 = -1}$$

A2 9/12

$$\begin{vmatrix} 1 & -2 & 1 \\ -1 & p-1 & 2 \\ 1 & -3 & p+2 \end{vmatrix} \overset{|\cdot(-1)}{=} \begin{vmatrix} 1 & -2 & 1 \\ 0 & p-3 & 3 \\ 0 & -1 & p+1 \end{vmatrix} = \begin{vmatrix} p-3 & 3 \\ -1 & p+1 \end{vmatrix} =$$

$$= (p+3)(p+1)+3 = \boxed{p^2-2p} = \boxed{p(p-2)}$$

$A_p \vec{x} = \vec{b}_p$ eind. lösbar $\Leftrightarrow \det(A_p) \neq 0 \Leftrightarrow p(p-2) \neq 0$

$\Leftrightarrow \boxed{p \neq 0} \vee \boxed{p \neq -2}$

$$\begin{vmatrix} 1-p & -2 & 1 \\ 2 & p-1 & 2 \\ 0 & -3 & p+2 \end{vmatrix} = (1-p)\begin{vmatrix} p-1 & 2 \\ -3 & p+2 \end{vmatrix} - 2\begin{vmatrix} -2 & 1 \\ -3 & p+2 \end{vmatrix} =$$

$$= (1-p)\big((p-1)(p+2)+6\big) - 2(-2p-4+3) =$$

$$= (1-p)(p^2+p+4) + 4p+2 = \boxed{-p^3+p+6}$$

$$\Rightarrow \quad x_1 = \frac{-p^3+p+6}{p(p-2)} = \frac{(-p^2-2p-3)\cancel{(p-2)}}{p\cancel{(p-2)}}$$

$p=2$ ist NS von $-p^3+p+6$

Horner:

	-1	0	1	6
2	$*$	-2	-4	6
	-1	-2	-3	0

$$= \boxed{\frac{p^2+2p+3}{p}}$$

A2 b)

7

$$A_{p,q}^T = \begin{pmatrix} p & 0 & 0 \\ 0 & 1 & 2 \\ 0 & q & -1 \end{pmatrix}$$

$$\begin{array}{c|c} & \begin{matrix} p & 0 & 0 \\ 0 & 1 & 2 \\ 0 & q & -1 \end{matrix} \\ \hline \begin{matrix} p & 0 & 0 \\ 0 & 1 & q \\ 0 & 2 & -1 \end{matrix} & \begin{matrix} p^2 & 0 & 0 \\ 0 & 1+q^2 & 2-q \\ 0 & 2-q & 5 \end{matrix} \end{array} \Rightarrow$$

$$A_{p,q} \cdot A_{p,q}^T = B$$

$$\Leftrightarrow \quad (1)\ \boxed{p^2 = 4}$$

$$\wedge \quad (2)\ \boxed{1+q^2 = 2}$$

$$\wedge \quad (3)\ \boxed{2-q = 3}$$

$$(3) \Rightarrow (4)\ \boxed{q = -1} \qquad (1) \Rightarrow \boxed{|p| = 2}$$

<u>Insgesamt:</u> $\boxed{p = \pm 2}$; $\boxed{q = -1}$

c)

12

$$\begin{vmatrix} 4 & -1 & 9-a \\ 2 & -3-a & -1 \\ 3-a & 2 & 4 \end{vmatrix} \,\big|\,(-2) = \begin{vmatrix} 0 & 5+2a & 11-a \\ 2 & -3-a & -1 \\ 3-a & 2 & 4 \end{vmatrix} =$$

$$= -2 \begin{vmatrix} 5+2a & 11-a \\ 2 & 4 \end{vmatrix} + (3-a) \begin{vmatrix} 5+2a & 11-a \\ -3-a & -1 \end{vmatrix} =$$

$$= -2(20+8a-22+2a) + (3-a)\big((-5-2a) + (3+a)(11-a)\big)$$

$$= -2(10a-2) + (3-a)(-5-2a+33-3a+11a-a^2) =$$

$$= -20a+4-3a^2+18a+84+a^3-6a^2-28a$$

$$= a^3 - 9a^2 - 30a + 88$$

$A_a \vec{x} = \vec{0}$ besitzt nichttriviale Lsgen.

$\Leftrightarrow \det(A_a) = 0 \Leftrightarrow a^3 - 9a^2 - 30a + 88 = 0$ (*)

$a = 2$ (raten!) ist NS von (*).

$$\begin{array}{r|rrrr} & 1 & -9 & -30 & 88 \\ 2) & * & 2 & -14 & -88 \\ \hline & 1 & -7 & -44 & 0 \end{array}$$

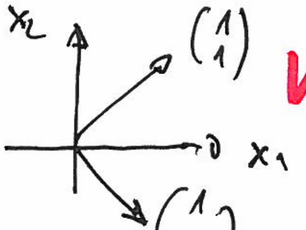
$\Rightarrow a^2 - 7a - 44 = 0 \Rightarrow a_{2/3} = \dfrac{7 \pm \sqrt{\overbrace{49+176}^{15}}}{2}$

$\Rightarrow a_2 = 11, \quad a_3 = -4$

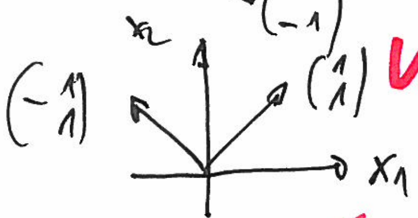
insgesamt: $\boxed{a_1 = 2} \vee \boxed{a_2 = 11} \vee \boxed{a_3 = -4}$

<u>A3</u> a) 2 $[f_1] = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad [f_2] = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

b) 4



f_1: Spiegelung a. d. x_1-Achse



f_2: Spiegelung a. d. x_2-Achse

c) 3 $[f_1 \circ f_2] = [f_1] \cdot [f_2] = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

d) 5 $\begin{pmatrix} \cos\varphi & -\sin\varphi \\ \sin\varphi & \cos\varphi \end{pmatrix}$ Drehmatrix. Beh. $[f_1 \circ f_2]$ ist eine Drehmatrix.

Es gilt: $[f_1 \circ f_2] \cdot [f_1 \circ f_2]^T = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = E$

und $\det([f_1 \circ f_2]) = 1 \Rightarrow$ Drehung um 0

Drehwinkel: $\cos\varphi = -1 \Rightarrow \boxed{\varphi = 180°}$ mit Drehwinkel $\varphi = 180°$.

e) 5

Rot. um $60°$: $[f_1] = \begin{pmatrix} \cos 60° & -\sin 60° \\ \sin 60° & \cos 60° \end{pmatrix} = \begin{pmatrix} \frac{1}{2} & -\frac{1}{2}\sqrt{3} \\ \frac{1}{2}\sqrt{3} & \frac{1}{2} \end{pmatrix}$ ✓

Orth. proj auf x-Achse: $[f_2] = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ ✓

Spiegelung an $y = x$: $[f_3] = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ✓

$\Rightarrow [f_3 \circ f_2 \circ f_1] = [f_3] \cdot [f_2] \cdot [f_1] = \begin{pmatrix} 0 & 0 \\ \frac{1}{2} & -\frac{1}{2}\sqrt{3} \end{pmatrix}$ ✓✓

f) 11

$p(x) = c_0 + c_1(x-0) + c_2(x-0)(x-1) + c_3(x-0)(x-1)(x-2)$ ✓

$x=0$: $c_0 = 1$

$x=1$: $c_0 + c_1 = 2$ ✓✓

$x=2$: $c_0 + 2c_1 + 2c_2 = 0$

$x=3$: $c_0 + 3c_1 + 6c_2 + 6c_3 = 1$

$\Rightarrow \boxed{c_0 = 1}$ ✓ $\boxed{c_1 = 1}$ ✓ $\boxed{c_2 = -\frac{3}{2}}$ ✓

$\boxed{c_3 = 1}$ ✓

$\Rightarrow p(x) = 1 + x - \frac{3}{2}x(x-1) + x(x-1)(x-2)$ ✓

$= 1 + x - \frac{3}{2}x^2 + \frac{3}{2}x + x(x^2 - 3x + 2)$ ✓

$= \boxed{x^3 - \frac{9}{2}x^2 + \frac{9}{2}x + 1}$ ✓✓