$\boxed{A1}$ (4P)

a) $|\vec{c}| = |\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}| \sin\varphi \overset{!}{=} |\vec{a}|$ $\frac{1}{2}$

$\underset{|\vec{a}| \neq 0}{\Longrightarrow} |\vec{b}| \sin\varphi = 1$ $\frac{1}{2}$ $\underset{|\vec{b}| = \sqrt{2}}{\Longrightarrow} \sqrt{2} \sin\varphi = 1$ $\frac{1}{2}$

$\Longrightarrow \sin\varphi = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$ $\frac{1}{2}$

$\Longrightarrow \boxed{\varphi = 45° \mathrel{\hat{=}} \frac{\pi}{4}}$ 1 oder $\boxed{\varphi = 135° \mathrel{\hat{=}} \frac{3}{4}\pi}$ 1

b) (4P)

$\vec{b}_{\parallel} = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}|^2} \vec{a}$ $1/2$ $= \frac{10}{5} \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix} = 2\vec{a} = \begin{pmatrix} 2 \\ 0 \\ 4 \end{pmatrix}$ $1/2$ $\qquad \vec{a} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}$

$\Longrightarrow \vec{b} = \vec{b}_{\parallel} + \begin{pmatrix} -3 \\ 1 \\ 1{,}5 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 5{,}5 \end{pmatrix}$ 1

$\Longrightarrow \vec{s} = \vec{a} + \vec{b} = \begin{pmatrix} 0 \\ 1 \\ 7{,}5 \end{pmatrix}$ 1 $\quad \vec{d} = \vec{a} - \vec{b} = \begin{pmatrix} 2 \\ -1 \\ -3{,}5 \end{pmatrix}$ 1

c) (4P)

$\vec{c} = \lambda_1 \vec{a} + \lambda_2 \vec{b} \Longrightarrow \lambda_1 \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + \lambda_2 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$

$\Longrightarrow$ (1) $2\lambda_1 + \lambda_2 = 4$ $1/2$

(2) $\lambda_2 = 2$ $1/2$ $\qquad$ (2) in (1): $\Longrightarrow$ (4) $\boxed{\lambda_1 = 1}$ $1/2$

(3) $-\lambda_1 + 2\lambda_2 = 3$ $1/2$

(4), (2) in (3): $3 = -1 + 4$ ✓ 1 $\Longrightarrow 1 \cdot \begin{pmatrix} 2 \\ 0 \\ -1 \end{pmatrix} + 2 \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \\ 3 \end{pmatrix}$ 1

d) $\vec{a} \perp \vec{b} \iff \vec{a} \cdot \vec{b} = 0$

(5P)

$$\iff (2\vec{p} + \vec{q})(-4\vec{p} + 5\vec{q}) = 0$$

$$\iff -8|\vec{p}|^2 + 10\vec{p}\vec{q} - 4\vec{p}\vec{q} + 5|\vec{q}|^2 = 0 \quad 1$$

$$\iff -8|\vec{p}|^2 + 6\vec{p}\vec{q} + 5|\vec{q}|^2 = 0 \quad 1$$

$$\underset{|\vec{p}|=|\vec{q}|}{\iff} -3|\vec{p}|^2 + \underset{\frac{1}{2}}{6\vec{p}\vec{q}} = 0 \iff \boxed{\vec{p}\vec{q} = \frac{1}{2}|\vec{p}|^2} \quad \frac{1}{2}$$

$$\Rightarrow \cos \sphericalangle(\vec{p}, \vec{q}) = \cos\alpha = \overset{\frac{1}{2}}{\frac{\vec{p}\cdot\vec{q}}{|\vec{p}||\vec{q}|}} = \frac{\vec{p}\cdot\vec{q}}{|\vec{p}|^2} \quad \frac{1}{2}$$

$$\Rightarrow \cos \sphericalangle(\vec{p}, \vec{q}) = \frac{1}{2} \Rightarrow \boxed{\alpha = 60^\circ \mathrel{\hat{=}} \frac{\pi}{3}} \quad 1$$

$\boxed{A2}$

a) (3P)

$$[f_3] = \begin{pmatrix} \cos 120^\circ & -\sin 120^\circ & 0 \\ \sin 120^\circ & \cos 120^\circ & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad 1$$

$$[f_2] = \begin{pmatrix} \cos 90^\circ & 0 & \sin 90^\circ \\ 0 & 1 & 0 \\ -\sin 90^\circ & 0 & \cos 90^\circ \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix} \quad 1$$

$$[f_1] = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 45^\circ & -\sin 45^\circ \\ 0 & \sin 45^\circ & \cos 45^\circ \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{pmatrix} \quad 1$$

b) (3P) $[f_3 \circ f_2 \circ f_1] = [f_3] \cdot [f_2] \cdot [f_1]$

Falk'sches Schema:

$$\begin{array}{ccc|ccc|ccc}
 & & & 0 & 0 & 1 & 1 & 0 & 0 \\
 & & & 0 & 1 & 0 & 0 & \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\
 & & & -1 & 0 & 0 & 0 & \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\
\hline
-\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 & 0 & -\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & \left(-\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}\right) & \left(\frac{\sqrt{6}}{4}-\frac{\sqrt{2}}{4}\right) \\
\frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 & 0 & -\frac{1}{2} & \frac{\sqrt{3}}{2} & 0 & \left(-\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}\right) & \left(\frac{\sqrt{2}}{4}+\frac{\sqrt{6}}{4}\right) \\
0 & 0 & 1 & -1 & 0 & 0 & -1 & 0 & 0
\end{array}$$

1,5 1,5

c) (7P)

– Die verkettete Fkt. $f_3 \circ f_2 \circ f_1$ ist eine LOT, da f_3, f_2, f_1 eine LOT.

$\frac{1}{2}$ Die Verkettung von LOT's ist wieder eine LOT

– Die verkettete Fkt ist eine <u>Drehung</u>

Es gilt:

$$[f_3 \circ f_2 \circ f_1]^T [f_3 \circ f_2 \circ f_1] = [f_1]^T [f_2]^T [f_3]^T [f_3][f_2][f_1] = \underline{E} = \underline{1}_3$$

Außerdem gilt

$\frac{1}{2}$ $$\det([f_3 \circ f_2 \circ f_1]) = \underbrace{\det([f_3])}_{=1} \cdot \underbrace{\det([f_2])}_{=1} \cdot \underbrace{\det([f_1])}_{=1} = 1$$

D.h. $f_3 \circ f_2 \circ f_1$ ist eine LOT $\wedge \det([f_3 \circ f_2 \circ f_1]) = 1$

Fortsetzung Blatt -5-

Drehachse:

$[f_3 \circ f_2 \circ f_1] =: \underline{A} \Rightarrow \underline{A}\vec{x} = \vec{x} \iff (\underline{A} - \underline{E})\vec{x} = \vec{0}$ $\frac{1}{2}$

$$\Rightarrow \begin{array}{cl} \left(\begin{array}{ccc|c} -1 & \left(-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right) & \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} & 0 \\ 0 & \left(-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} - 1\right) & \left(\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}\right) & 0 \\ -1 & 0 & -1 & 0 \end{array}\right) & \begin{array}{l} (E_1) \\ (E_2) \quad \text{Normalenv. } \vec{n}_2 \\ (E_3) \quad \text{Normalenv. } \vec{n}_3 \end{array} \end{array}$$

$\vec{n}_2 \times \vec{n}_3 = \vec{x} =: \vec{r}$:

$$\vec{r} = \begin{pmatrix} \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} + 1 \\ -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \\ -\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} - 1 \end{pmatrix} \quad \frac{1}{2}$$

Suche Vektor $\vec{s}$ mit $\vec{s} \perp \vec{r}$:

Wähle $\vec{s} = \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}$ $\frac{1}{2}$

$$\Rightarrow f(\vec{s}) = \begin{pmatrix} 0 & \left(-\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right) & \left(\frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}\right) \\ 0 & \left(-\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}\right) & \left(\frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4}\right) \\ -1 & 0 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} \\ \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \\ -1 \end{pmatrix}$$

$\frac{1}{2}$ $\frac{1}{2}$

Drehwinkel:

$$\cos \sphericalangle(\vec{s}, f(\vec{s})) = \cos\alpha = \frac{\vec{s} \cdot f(\vec{s})}{|\vec{s}|\,|f(\vec{s})|}$$

$$\vec{s} \cdot f(\vec{s}) = \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} - 1 \approx -0{,}74 \quad \tfrac{1}{2}$$

$$|\vec{s}| = \sqrt{2} \approx 1{,}41 \quad \tfrac{1}{2}; \quad f(\vec{s}) \approx \sqrt{0{,}07 + 0{,}93 + 1} \approx 1{,}41 \quad \tfrac{1}{2}$$

$$\Rightarrow \cos\alpha = \frac{-0{,}74}{1{,}41 \cdot 1{,}41} \Rightarrow \boxed{\alpha = 111{,}85^\circ} \quad 1$$

Orientierung:

$$\vec{\alpha}_0 = \frac{\vec{s} \times f(\vec{s})}{|\vec{s} \times f(\vec{s})|} \approx \frac{1}{\sqrt{3{,}47}} \begin{pmatrix} -\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} \\ \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4} + 1 \\ \frac{\sqrt{2}}{4} + \frac{\sqrt{6}}{4} \end{pmatrix} \approx \frac{1}{\sqrt{3{,}47}} \begin{pmatrix} -0{,}97 \\ 1{,}26 \\ 0{,}97 \end{pmatrix} \quad 1$$