$$\int \sqrt{3+2x-x^2}\,dx = \int \sqrt{-(x^2-2x+1)+3+1}\,dx$$

$$= \int \sqrt{4-(x-1)^2}\,dx \underset{\boxed{\begin{array}{l} z := x-1 \\ dz = dx \end{array}}}{=} \int \sqrt{4-z^2}\,dz \underset{\boxed{\begin{array}{l} z := 2\sin t \\ dz = 2\cos t \end{array}}}{=}$$

$$= \int \sqrt{4-4\sin^2 t}\cdot 2\cos t\,dt = 2\int \sqrt{4(1-\sin^2 t)}\cdot \cos t\,dt =$$

$$= 4\int \cos^2 t\,dt \underset{\text{Formel-sammlung}}{=} 4\left[\frac{1}{2}t + \frac{1}{4}\sin(2t)\right] + C_1 =$$

$$= 2t + \sin(2t) + C_1 = 2t + 2\sin t\cos t + C_1 = 2t + 2\sin t\sqrt{1-\sin^2 t} + C_1$$

$$\underset{\substack{\boxed{t = \arcsin\left(\frac{z}{2}\right)} \\ \text{R.S.}}}{=} 2\arcsin\left(\frac{z}{2}\right) + \sqrt{1-\left[\sin\left(\arcsin\left(\frac{z}{2}\right)\right)\right]^2} + C_2$$

$$= 2\arcsin\left(\frac{z}{2}\right) + \sqrt{1-\left(\frac{z}{2}\right)^2} + C_2 = 2\arcsin\left(\frac{z}{2}\right) + \sqrt{1-\frac{z^2}{4}} + C_2$$

$$= \sqrt{\frac{4-z^2}{4}} + 2\arcsin\left(\frac{z}{2}\right) + C_2 = \frac{1}{2}\sqrt{4-z^2} + 2\arcsin\left(\frac{z}{2}\right) + C_2$$

$$\underset{\substack{\boxed{z = x-1} \\ \text{R.S.}}}{=} \frac{1}{2}\sqrt{4-(x-1)^2} + 2\arcsin\left(\frac{x-1}{2}\right) + C_3$$

$$= \frac{1}{2}\sqrt{4-(x^2-2x+1)} + 2\arcsin\left(\frac{x-1}{2}\right) + C_3$$

$$= \boxed{\frac{1}{2}\sqrt{3+2x-x^2} + 2\arcsin\left(\frac{x-1}{2}\right) + C_3}$$