$\boxed{A8}$ $y'' + 6y' + cy = 0$

$\Rightarrow$ <u>char. Poly.:</u> $\lambda^2 + 6\lambda + c = 0$

$$\lambda_{1/2} = \frac{-6 \pm \sqrt{36-4c}}{2}$$

a) $\boxed{c=5}$: $\lambda_{1/2} = \dfrac{-6 \pm \overset{4}{\sqrt{36-20}}}{2} \Rightarrow \lambda_1 = -1,\ \lambda_2 = -5$

$\Rightarrow$ <u>F.S.:</u> $\{e^{-x}, e^{-5x}\} \Rightarrow y_H = C_1 e^{-x} + C_2 e^{-5x}$

b) $\boxed{c=9}$: $\lambda_{1/2} = \dfrac{-6 \pm \sqrt{36-36}}{2} \Rightarrow \lambda_1 = \lambda_2 = -3$

$\Rightarrow$ <u>F.S.:</u> $\{e^{-3x}, xe^{-3x}\} \Rightarrow y_H = C_1 e^{-3x} + C_2 x e^{-3x}$

c) $\boxed{c=13}$: $\lambda_{1/2} = \dfrac{-6 \pm \sqrt{36-52}}{2} \Rightarrow \lambda_{1/2} = -3 \pm 2i$

$\Rightarrow$ <u>F.S.:</u> $\{e^{-3x}\cos 2x, e^{-3x}\sin 2x\}$

$\Rightarrow y_H = e^{-3x}(C_1 \cos 2x + C_2 \sin 2x)$

9) a) $\ddot{s}+2\dot{s}+2s=0$; Ansatz: $s=e^{\lambda t}$

$\Rightarrow$ char. Poly.: $\lambda^2+2\lambda+2=0 \Rightarrow \lambda_{1/2}=-1\pm i$

$\Rightarrow$ F.S.: $\{e^{-x}\cos x,\ e^{-x}\sin x\}$

$\Rightarrow$ allg. Lsg. $s_h = e^{-x}(C_1\cos x + C_2\sin x)$

$\Rightarrow s_h' = -e^{-x}(C_1\cos x + C_2\sin x) + e^{-x}(-C_1\sin x + C_2\cos x)$

$s_h(0)=1$; $s_h(0)=C_1 \stackrel{!}{=} 1 \Rightarrow C_1=1$

$s_h'(0)=1$; $s_h'(0)=-1(1)+1\cdot C_2 \stackrel{!}{=} 1 \Rightarrow C_2=2$

$\Rightarrow s_p(t)=e^{-t}(\cos t + 2\sin t)$ (spez. Lsg. d. AWP)

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b) $y''+4y'+(4+\omega^2)y=0 \Rightarrow$ char. Poly.: $\lambda^2+4\lambda+4+\omega^2=0$

$\Rightarrow \lambda_{1/2}=\dfrac{-4\pm\sqrt{-4\omega^2}}{2} \Rightarrow \lambda_{1/2}=-2\pm\omega i$

$\Rightarrow$ F.S.: $\{e^{-2x}\cos\omega x,\ e^{-2x}\sin\omega x\}$

$\Rightarrow$ allg. Lsg.: $y_h=e^{-2x}(C_1\cos\omega x + C_2\sin\omega x)$

$\Rightarrow y_h' = -2e^{-2x}(C_1\cos\omega x + C_2\sin\omega x) + e^{-2x}(-C_1\omega\sin\omega x + C_2\omega\cos\omega x)$

$y_h(0)=1$; $y_h(0)=C_1 \stackrel{!}{=} 1 \Rightarrow C_1=1$

$y_h'(0)=\omega-2$; $y_h'(0)=-2(1)+(C_2\omega) \stackrel{!}{=} \omega-2$

$\Rightarrow C_2=1$

$\Rightarrow y_p(x)=e^{-2x}(\cos\omega x+\sin\omega x)$ (spez. Lsg. d. AWP)

$\boxed{9c)}$ $y'' - 2ky' + k^2 y = 0 \Rightarrow$ char. P.: $\lambda^2 - 2k\lambda + k^2 = 0$

$\Rightarrow \lambda_{1/2} = \dfrac{2k \pm \sqrt{4k^2 - 4k^2}}{2} \Rightarrow \lambda_1 = \lambda_2 = \lambda = k$

$\Rightarrow$ <u>doppelte</u> <u>NS</u> $\Rightarrow$ F.S.: $\{e^{kx}, xe^{kx}\}$

$\Rightarrow$ <u>allg. Lsg.</u>: $y_h = (C_1 + C_2 x)e^{kx}$

$\Rightarrow y_h' = C_2 e^{kx} + (C_1 + C_2 x)e^{kx} \cdot k$

$y_h(0) = \sqrt{2}$

$y_h(0) = C_1 \overset{!}{=} \sqrt{2} \Rightarrow C_1 = \sqrt{2}$

$y_h'(0) = k\sqrt{2}$

$y_h'(0) = C_2 + \sqrt{2}k \overset{!}{=} k\sqrt{2} \Rightarrow C_2 = 0$

$\Rightarrow y(x) = \sqrt{2}e^{kx}$ (spez. Lsg. d. AWP)

$\boxed{A10}$ a) $\{e^{2x}, e^{-4x}\}$

$\Rightarrow \lambda_1 = 2,\ \lambda_2 = -4$ sind EW

$\Rightarrow$ <u>char. Poly.</u>: $(\lambda-2)(\lambda+4) = 0 \Rightarrow \lambda^2 + 2\lambda - 8 = 0$

$\Rightarrow$ DGL: $y'' + 2y' - 8y = 0$

---

b) $\{\cos 4x, \sin 4x\} \Rightarrow \lambda_1 = 4i,\ \lambda_2 = -4i$ sind EW

$\Rightarrow$ <u>char. Polyn.</u>: $(\lambda - 4i)(\lambda + 4i) = 0 \Leftrightarrow \lambda^2 + 16 = 0$

$\Rightarrow$ DGL: $y'' + 16y = 0$

---

c) $\{e^{2x}, x e^{2x}\} \Rightarrow \lambda_1 = \lambda_2 = 2$ sind doppelte reelle EW

$\Rightarrow (\lambda - 2)^2 = 0 \Leftrightarrow \lambda^2 - 4\lambda + 4 = 0$

$\Rightarrow$ DGL: $y'' - 4y' + 4y = 0$

---

d) $\{e^{-x}\cos 3x, e^{-x}\sin 3x\}$

$\Rightarrow \lambda_{1/2} = -1 \pm 3i$

$\Rightarrow$ <u>char. Polyn.</u>: $(\lambda + 1 - 3i)(\lambda + 1 + 3i) = 0$

$\Leftrightarrow \lambda^2 - 2\lambda + 10 = 0$

$\Rightarrow$ DGL: $y'' - 2y' + 10y = 0$

$\boxed{A11}$ a) $y'' - y' - 2y = 2x^2 - e^{2x}$

hom. DGL: $y'' - y' - 2y = 0 \Rightarrow \lambda^2 - \lambda - 2 = 0$

$\Rightarrow \lambda_{1/2} = \frac{1 \pm \sqrt{1+8}^{\,3}}{2} \Rightarrow \lambda_1 = 2, \quad \lambda_2 = -1$

$\Rightarrow y_{hom} = C_1 e^{2x} + C_2 e^{-x}$

$\Longrightarrow$ Resonanz-ansatz

$y_p = Axe^{2x} + B + Cx + Dx^2$

$y_p' = Ae^{2x} + 2Axe^{2x} + C + 2Dx$

$y_p'' = 2Ae^{2x} + 2Ae^{2x} + 2Axe^{2x} \cdot 2 + 2D$

$\Longrightarrow$ einsetzen in hom. DGL

$4Ae^{2x} + \cancel{4Axe^{2x}} + 2D - Ae^{2x} - \cancel{2Axe^{2x}} - C - 2Dx$
$- \cancel{2Axe^{2x}} - 2B - 2Cx - 2Dx^2 = 2x^2 - e^{2x}$

$\Longrightarrow 3Ae^{2x} + x(-2D - 2C) + 2D - C - 2B - 2Dx^2 = 2x^2 - e^{2x}$

$\Longrightarrow$ Koeff.vergl.

$3A = -1 \Rightarrow A = -\frac{1}{3}$

$-2D = 2 \Rightarrow D = -1$

$-2D - 2C = 0 \Rightarrow C = 1$

$2D - C - 2B = 0 \Rightarrow -2B = 2 + 1 \Rightarrow B = -\frac{3}{2}$

$\Rightarrow y_{all} = y_{Hom} + y_p$

$\Rightarrow \boxed{y_{all} = C_1 e^{2x} + C_2 e^{-x} - \frac{1}{3}xe^{2x} - \frac{3}{2} + x - x^2}$

$\boxed{11\ b}$ $y'' - y' - 2y = \sin x$

$\Rightarrow y_{hom} = C_1 e^{2x} + C_2 e^{-x}$

Ansatz für inh. DGL: $y_p = A_1 \cos x + A_2 \sin x$

$\Rightarrow y_p' = -A_1 \sin x + A_2 \cos x$

$\Rightarrow y_p'' = -A_1 \cos x - A_2 \sin x$

$\xRightarrow{\text{einsetzen}}$ $-A_1 \cos x - A_2 \sin x + A_1 \sin x - A_2 \cos x$
$- 2A_1 \cos x - 2A_2 \sin x = \sin x$

$\Rightarrow -3A_1 \cos x - 3A_2 \sin x + A_1 \sin x - A_2 \cos x = \sin x$

$\Rightarrow \cos x(-3A_1 - A_2) + \sin x\,(A_1 - 3A_2) = \sin x$

$$\Rightarrow \left.\begin{aligned} -3A_1 - A_2 &= 0 \\ A_1 - 3A_2 &= 1 \end{aligned}\right\} \Rightarrow \left(\begin{array}{cc|c} -3 & -1 & 0 \\ 1 & -3 & 1 \end{array}\right) \begin{array}{l} \\ |\cdot 3 \end{array}$$

$$\rightarrow \left(\begin{array}{cc|c} -3 & -1 & 0 \\ 0 & -10 & 3 \end{array}\right) \Rightarrow A_2 = -\frac{3}{10}$$

$$\Rightarrow A_1 = \frac{1}{10}$$

$$\Rightarrow \boxed{y_{all} = C_1 e^{2x} + C_2 e^{-x} + \frac{1}{10} \cos x - \frac{3}{10} \sin x}$$

$\boxed{11c}$ $y'' - y' - 2y = e^{-x}$

$\Rightarrow y_{Hom} = C_1 e^{2x} + C_2 e^{-x}$

$\Longrightarrow$ Resonanz-ansatz

$y_p = Axe^{-x}$

$y_p' = Ae^{-x} - Axe^{-x}$

$y_p'' = -Ae^{-x} - Ae^{-x} + Axe^{-x}$

$\Rightarrow$ einsetzen

$-Ae^{-x} - Ae^{-x} + Axe^{-x} - Ae^{-x} + Axe^{-x} - 2Axe^{-x} = e^{-x}$

$-3Ae^{-x} = e^{-x} \Rightarrow -3A = 1 \Rightarrow A = -\frac{1}{3}$

$\Rightarrow \boxed{y_{all} = C_1 e^{2x} + C_2 e^{-x} - \frac{1}{3}xe^{-x}}$

$\boxed{A12}$ a) $y'' + 2y' + 5y = 50x + 8e^{-x}$ (*)

<u>1. Lsg. d. hom. DGL</u>: $y'' + 2y' + 5\lambda = 0$

$\Rightarrow$ <u>char. Pol.</u>: $\lambda^2 + 2\lambda + 5 = 0 \Rightarrow \lambda_{1/2} = \dfrac{-2 \pm \sqrt{4-20}}{2}$

$\Rightarrow \lambda_{1/2} = -1 \pm 2i \Rightarrow$ <u>F.S.</u>: $\{e^{-x}\cos 2x, e^{-x}\sin 2x\}$

$\Rightarrow y_h = (C_1 \cos 2x + C_2 \sin 2x) e^{-x}$

<u>2. Lsg. d. inh. DGL</u>: $r(x) = 50x + 8e^{-x}$

<u>Störansatz</u>: $y_p = A_0 + A_1 x + A_2 e^{-x} \Rightarrow y_p' = A_1 - A_2 e^{-x}$

$y_p'' = A_2 e^{-x}$. Einsetzen in (*):

$\underline{A_2 e^{-x}} + \underline{2A_1} - 2\underline{A_2 e^{-x}} \cancel{50x+} + 5\underline{A_0} + 5\underline{A_1 x} + 5\underline{A_2 e^{-x}} = 50x + 8e^{-x}$

$\Rightarrow e^{-x}(A_2 - 2A_2 + 5A_2) + 5A_1 x + 2A_1 + 5A_0 = 50x + 8e^{-x}$

$\Rightarrow \quad 4A_2 = 8 \Rightarrow A_2 = 2; \quad 5A_1 = 50 \Rightarrow A_1 = 10$

$\Rightarrow \quad 2A_1 + 5A_0 = 0 \Rightarrow 5A_0 = -20 \Rightarrow A_0 = -4$

$\Rightarrow y_{all} = y_H + y_p \Rightarrow y_{all} = e^{-x}(C_1 \cos 2x + C_2 \sin 2x + 2) + 10x - 4$

$\boxed{\text{A12c}}$ $\quad y''' + y = 12\cosh x = \frac{12}{2}\left(e^{x}+e^{-x}\right) = 6\left(e^{x}+e^{-x}\right)$

hom. DGL: $\quad y''' + y = 0 \Rightarrow$ char. Pol.: $\quad \lambda^3 + 1 = 0$

$\Rightarrow \quad \lambda = \sqrt[3]{-1}$, es gilt: $\quad z_k = \sqrt[3]{-1} = e^{i\frac{(\pi + k\cdot 2\pi)}{3}} \quad (k=0,1,2)$

$\Rightarrow z_0 = e^{i\frac{\pi}{3}} = \cos\frac{\pi}{3} + i\sin\frac{\pi}{3} = \frac{1}{2} + \frac{i}{2}\sqrt{3}$

$z_1 = e^{i\pi} = \cos\pi + i\sin\pi = -1$

$z_2 = e^{i\frac{5}{3}\pi} = \cos\frac{5}{3}\pi + i\sin\frac{5}{3}\pi = \frac{1}{2} - \frac{i}{2}\sqrt{3}$

$$\Rightarrow \boxed{y_{hom} = C_0 e^{-x} + e^{\frac{x}{2}}\left(C_1\cos\frac{\sqrt{3}}{2}x + C_2\sin\frac{\sqrt{3}}{2}x\right)}$$

inhom. DGL: <u>Resonanzansatz:</u>

$y_p = Ae^{x} + Be^{-x}\cdot x$

$y_p' = Ae^{x} + Be^{-x} + Bxe^{-x}(-1)$

$y_p'' = Ae^{x} - Be^{-x} - Be^{-x} + Bxe^{-x}$

$\underset{\text{einsetzen}}{\Rightarrow} \quad y_p''' = Ae^{x} + Be^{-x} + Be^{-x} + Be^{-x} - Bxe^{-x}$

$= Ae^{x} + 3Be^{-x} - Bxe^{-x}$

$\underset{\text{einsetzen}}{\Longrightarrow} \quad Ae^{x} + 3Be^{-x} - Bxe^{-x} + Ae^{x} + Bxe^{-x} = 6e^{x} + 6e^{-x}$

$\Rightarrow \quad 2Ae^{x} + e^{-x}\,3B = 6e^{x} + 6e^{-x}$

$\Rightarrow \quad 2A = 6 \Rightarrow A = 3\,; \quad 3B = 6 \Rightarrow B = 2$

$$\Rightarrow \boxed{y_{all} = y_{hom} + 3e^{x} + 2xe^{-x}}$$

$\boxed{\text{A12d}}$ $y^{(4)} - 3y'' - 4y = x^2 + e^{-x}$

hom. DGL: $\lambda^4 - 3\lambda^2 - 4 = 0$ (biquadr. Gl.)

$w := \lambda^2 \Rightarrow w^2 - 3w - 4 = 0 \Rightarrow w_1 = 4,\ w_2 = -1$

$\Rightarrow \lambda_1 = 2,\ \lambda_2 = -2,\ \lambda_3 = i,\ \lambda_4 = -i$

$$\Rightarrow \boxed{y_{hom} = C_1 e^{2x} + C_2 e^{-2x} + C_3 \cos x + C_4 \sin x}$$

inhom. DGL: Ansatz:

$$y_p = A + Bx + Cx^2 + De^{-x}$$
$$y_p' = B + 2Cx - De^{-x}$$
$$y_p'' = 2C + De^{-x}$$
$$y_p''' = -De^{-x}$$
$$y_p^{(4)} = De^{-x}$$

$\Longrightarrow$ einsetzen

$\cancel{A + Bx + Cx^2 + De^{-x} - 6C - 6De^{-x}}$

$\cancel{A + Bx + Cx^2 + De^{-x} - 6C - 3De^{-x}}$

$De^{-x} - 6C - 3De^{-x} - 4A - 4Bx - 4Cx^2 - 4De^{-x} = x^2 + e^{-x}$

$\Rightarrow -6De^{-x} - 4Bx - 6C - 4A - 4Cx^2 = x^2 + e^{-x}$

Koeff.-vergl.

$\Rightarrow -6D \overset{!}{=} 1 \Rightarrow \boxed{D = -\frac{1}{6}}$

$-4C \overset{!}{=} 1 \Rightarrow \boxed{C = -\frac{1}{4}}$

$-4B \overset{!}{=} 0 \Rightarrow \boxed{B = 0}$

$-6C - 4A \overset{!}{=} 0 \underset{C = -\frac{1}{4}}{\Longrightarrow} \frac{3}{2} - 4A = 0 \Rightarrow \boxed{A = \frac{3}{8}}$

$$\Rightarrow \boxed{y_{all} = y_{hom} + \frac{3}{8} - \frac{1}{4}x^2 - \frac{1}{6}e^{-x}}$$

$\boxed{A13}$ a) $y''' + 3y'' + 3y' + y = x^3 + e^{-x} \sin 2x$

$\Rightarrow$ char. Poly.: $\lambda^3 + 3\lambda^2 + 3\lambda + 1 = 0$

$\lambda_1 = -1$ ist NS $\Rightarrow$ POLDIV: $(\lambda + 1)(\lambda^2 + 2\lambda + 1) = 0$

$$\Rightarrow \lambda_{2/3} = \frac{-2 \pm \sqrt{4-4}}{2} \Rightarrow \lambda_2 = \lambda_3 = 1$$

$\Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = \lambda = 1$ ist 3-fache NS.

$$\Rightarrow y_h = (C_1 + C_2 x + C_3 x^2) e^{-x}$$

$\Rightarrow$ <u>Störansatz</u>: $y_p = A_0 + A_1 x + A_2 x^2 + A_3 x^3 + e^{-x}(B_1 \cos 2x + B_2 \sin 2x)$

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b) $y''' + 3y'' + 3y' + y = x^2 (e^x + e^{-x})$

$\underset{a)}{\Longrightarrow}$ $\lambda = -1$ ist 3-fache NS d. char. Poly; FS: $\{e^{-x}, xe^{-x}, x^2 e^{-x}\}$

$\Rightarrow$ <u>Störansatz</u>:

$$y_p = e^x (A_0 + A_1 x + A_2 x^2) + \underbrace{x^3 e^{-x} (B_0 + B_1 x + B_2 x^2)}$$

da $x^2 e^{-x}$ Lsg.
d. hom. DGL
und $\lambda = -1$ 3-fache NS.

$\boxed{A14}$ a) $y'' + 2y' - 3y = 2\sin x$ (*)

1. Lsg. d. hom. DGL: $y'' + 2y' - 3y = 0$

char. Poly.: $\lambda^2 + 2\lambda - 3 = 0 \Rightarrow \lambda_{1/2} = \frac{-2 \pm \sqrt{4+12}^{\,4}}{2}$

$\Rightarrow \lambda_1 = 1,\ \lambda_2 = -3 \Rightarrow y_h = C_1 e^{x} + C_2 e^{-3x}$

2. Lsg. d. inh. DGL: Störansatz: $\boxed{y_p = A_1 \cos x + A_2 \sin x}$

$\Rightarrow y_p' = -A_1 \sin x + A_2 \cos x,\ y_p'' = -A_1 \cos x - A_2 \sin x.$

Einsetzen in (*): $-A_1 \cos x - A_2 \sin x - 2A_1 \sin x + 2A_2 \cos x - 3A_1 \cos x - 3A_2 \sin x$
$= 2\sin x$

$\Rightarrow \sin x(-4A_2 - 2A_1) + \cos x(-4A_1 + 2A_2) = 2\sin x$

$\Rightarrow -4A_2 - 2A_1 = 2\ ;\ -4A_1 + 2A_2 = 0 \Rightarrow A_2 = 2A_1$

$\Rightarrow A_1 = -\frac{1}{5} \Rightarrow A_2 = -\frac{2}{5}$

$\Rightarrow y_{all} = y_H + y_p \Rightarrow y_{all} = C_1 e^{x} + C_2 e^{-3x} - \frac{1}{5}\cos x - \frac{2}{5}\sin x$

AWP: $y_{all}(0) = 0\ ;\ y_{all}'(0) = 1$ (Koeff. vergleich)

$y_{all}(0) = 0 \Rightarrow C_1 + C_2 - \frac{1}{5} \overset{!}{=} 0 \Rightarrow C_1 + C_2 = \frac{1}{5}$

$y_{all}'(0) = 1 \Rightarrow y_{all}' = C_1 e^{x} - 3C_2 e^{-3x} + \frac{1}{5}\sin x - \frac{2}{5}\cos x$

$y_{all}'(0) = C_1 - 3C_2 - \frac{2}{5} \overset{!}{=} 1 \Rightarrow C_1 = \frac{1}{2}\ ;\ C_2 = -\frac{3}{10}$

$\Rightarrow y_p(x) = -\frac{1}{5}\cos x - \frac{2}{5}\sin x + \frac{1}{2}e^{x} - \frac{3}{10}e^{-3x}$

(spez. Lsg. d. AWP)

$\boxed{14b}$ $y''+6y'+25y = 50x-13$ 8i

$$\Rightarrow \quad \lambda^2+6\lambda+25=0 \Rightarrow \lambda_{1/2} = \frac{-6 \pm \sqrt{36-100}}{2}$$

$$\Rightarrow \lambda_1 = -3+4i, \quad \lambda_2 = -3-4i$$

$$\Rightarrow y_{hom} = e^{-3x}\left(C_1 \cos 4x + C_2 \sin 4x\right)$$

normaler Ansatz $\Longrightarrow$

$$\left.\begin{array}{l} y_p = A + Bx \\ y_p' = B \\ y_p'' = 0 \end{array}\right\} \xrightarrow[\text{einsetzen}]{} 6B+25A+25Bx = 50x-13$$

$$\Rightarrow \begin{array}{l} 25B = 50 \Rightarrow B=2 \\ 6B+25A=-13 \Rightarrow 25A = -13-12 = -25 \Rightarrow A=-1 \end{array}$$

$$\Rightarrow y_{all} = e^{-3x}(C_1\cos 4x + C_2 \sin 4x) - 1 + 2x$$

$$y_{all}' = -3C_1 e^{-3x}\cos 4x + e^{-3x}C_1(-\sin 4x)\cdot 4 - e^{-3x}C_2 \sin 4x + C_2 e^{-3x}(\cos 4x)\cdot 4 + 2$$

$y_{all}(0) = 1$, $y_{all}(0) = C_1 - 1 \overset{!}{=} 1 \Rightarrow C_1 = 2$

$y_{all}'(0) = 0$, $y_{all}' = -3\cdot 2 + 4C_2 + 2 \overset{!}{=} 0 \Rightarrow C_2 = 1$

$$\Rightarrow \boxed{y_{speziell} = 2x-1+e^{-3x}(\sin 4x + 2\cos 4x)}$$

$\boxed{A15}$ a) $y'' + ay' + 2y = 0 \Rightarrow$ char. Poly.: $\lambda^2 + a\lambda + 2 = 0$

$$\Rightarrow \lambda_{1/2} = \frac{-a \pm \sqrt{a^2-8}}{2}$$

Die DGL soll $y = e^{-x} - e^{-2x}$ als Lsg besitzen; somit muß $\lambda_1 = -1$ und $\lambda_2 = -2$ ein EW d. char. P. sein:

$$\Rightarrow \left.\begin{aligned} \frac{-a+\sqrt{a^2-8}}{2} &= -1 \quad (1) \\ \frac{-a-\sqrt{a^2-8}}{2} &= -2 \quad (2) \end{aligned}\right\} \Rightarrow \quad (1)+(2): \quad -2a = -6 \Rightarrow \boxed{a=3}$$

$$\Rightarrow y_{all} = C_1 e^{-x} + C_2 e^{-2x}$$

b) $y'' + 2y' + 2y = g(x) \quad (*)$

$y_p = 3\sin x$ ist part. Lsg. $\Rightarrow y_p' = 3\cos x, \; y_p'' = -3\sin x$

Einsetzen in $(*)$: $-3\sin x + 6\cos x + 6\sin x = 6\cos x + 3\sin x = g(x)$

char. Poly.: $\lambda^2 + 2\lambda + 2 = 0 \Rightarrow \lambda_{1/2} = -1 \pm i$

$$\Rightarrow y_{all} = e^{-x}(C_1\cos x + C_2 \sin x)$$

Lsg. d. inh. DGL: Störansatz: $y_p = A_1\cos x + A_2 \sin x$

$y_p' = -A_1 \sin x + A_2\cos x$

$y_p'' = -A_1\cos x - A_2 \sin x$

$$\Rightarrow -A_1\cos x - A_2\sin x - 2A_1\sin x + 2A_2\cos x + 2A_1\cos x + 2A_2\sin x = 6\cos x + 3\sin x$$

$$\Rightarrow \cos x\,(A_1 + 2A_2) + \sin x\,(A_2 - 2A_1) = 6\cos x + 3\sin x$$

$$\Rightarrow \left.\begin{aligned} A_1 + 2A_2 &= 6 \\ A_2 - 2A_1 &= 3 \end{aligned}\right\} \Rightarrow \quad A_1 = 0, \; A_2 = 3$$

$$\Rightarrow y_{all} = e^{-x}(C_1\cos x + C_2\sin x) + 3\sin x$$

$$y_{all}' = -e^{-x}(C_1\cos x + C_2 \sin x) + e^{-x}(-C_1\sin x + C_2\cos x) + 3\cos x$$

$\boxed{A15}$ b) Fortsetzung:

$y_{all}(0) = 0$; $y_{all}(0) = c_1 \stackrel{!}{=} 0 \Rightarrow \boxed{c_1 = 0}$

$y'_{all}(0) = 5$; $y'_{all}(0) = -(c_1) + c_2 + 3 \stackrel{!}{=} 5 \Rightarrow \boxed{c_2 = 2}$

$\Rightarrow y_p = 2e^{-x}\sin x + 3\sin x$ (spez. Lsg. d. AWP)

$\boxed{A16}$ a) $\underline{z.z.}$: $\tilde{y} = e^{-x}\cos 2x$ ist eine spez. Lsg d. DGL

$y''' + y' - 10y = 0$ (*)

Bestimme: $\tilde{y}' = \dots$, $\tilde{y}'' = \dots$, $\tilde{y}''' = \dots$ einsetzen in (*).

Es gilt: $\lambda_{1/2} = -1 \pm 2i$ sind EW d. char. Polynoms: $\lambda^3 + \lambda - 10 = 0$

$\Rightarrow$ $(\lambda + 1 + 2i)(\lambda + 1 - 2i) = 0 \Leftrightarrow \lambda^2 + 2\lambda + 5 = 0$

$\underline{\text{POLDIV}}$: $(\lambda^3 + \lambda - 10) : (\lambda^2 + 2\lambda + 5) = \lambda - 2$

$\Rightarrow$ $\lambda_3 = 2$ ist ein weiterer EW.

$\Rightarrow$ $y_H = e^{-x}(C_1 \cos 2x + C_2 \sin 2x) + C_3 e^{2x}$

b) $y''' + y' - 10y = e^{-x}$; $\Rightarrow$ Störansatz: $y_p = A_1 e^{-x}$, $y_p' = -A_1 e^{-x}$,

$y_p'' = A_1 e^{-x}$, $y_p''' = -A_1 e^{-x}$

$\Rightarrow$ $e^{-x}(-A_1 - A_1 - 10A_1) = e^{-x} \Rightarrow A_1 = -\frac{1}{12}$

$\Rightarrow$ $y_{all} = -\frac{1}{12}e^{-x} + e^{-x}(C_1 \cos 2x + C_2 \sin 2x) + C_3 e^{2x} = y_H + y_p$

$y_{all}' = \frac{1}{12}e^{-x} - e^{-x}(C_1 \cos 2x + C_2 \sin 2x) + e^{-x}(-2C_1 \sin 2x + 2C_2 \cos 2x)$
$+ 2C_3 e^{2x}$

y_{all} ist beschr. für $x \to \infty \Rightarrow \boxed{C_3 = 0}$

$y_{all}(0) = 0$, $y_{all}'(0) = 1 \Rightarrow C_1 = \frac{1}{12}$, $C_2 = \frac{1}{2}$

$\Rightarrow$ $y_p = -\frac{1}{12}e^{-x} + \frac{1}{12}e^{-x}\cos 2x + \frac{1}{2}e^{-x}\sin 2x$

(spez. Lsg. d. AWP)