S. 87, A 22

(1) $\dot{x} + y = \sin 2t$ $\qquad$ $x(0) = 1, \quad y(0) = 0$

(2) $\dot{y} - x = \cos 2t$

(1) $\Rightarrow$ (3) $y = -\dot{x} + \sin 2t \Rightarrow$ (4) $\dot{y} = -\ddot{x} + 2\cos 2t$

(4) in (2) $\Rightarrow$ $-\ddot{x} + 2\cos 2t - x = \cos 2t$

$\Rightarrow$ $-\ddot{x} - x = -\cos 2t$

$\Rightarrow$ (5) $\boxed{\ddot{x} + x = \cos 2t}$

hom. DGL: $\quad \lambda^2 + 1 = 0 \Rightarrow \lambda_{1/2} = \pm j$

F.S. $\quad \{\sin t, \cos t\}$

$y_h = C_1 \sin t + C_2 \cos t$

inhom. DGL:

Ansatz
$$\left.\begin{aligned} y_p &= A_1 \cos 2t + A_2 \sin 2t \\ y_p' &= -2A_1 \sin 2t + 2A_2 \cos 2t \\ y_p'' &= -4A_1 \cos 2t - 4A_2 \sin 2t \end{aligned}\right\} (*)$$

(*) in (5): $-4A_1 \cos 2t - 4A_2 \sin 2t + A_1 \cos 2t + A_2 \sin 2t = \cos 2t$

$\Rightarrow$ $\cos 2t(-4A_1 + A_1) = \cos 2t$

$+ \sin 2t(-4A_2 + A_2)$

$$\Longrightarrow \quad \boxed{A_1 = -\frac{1}{3}}\ , \quad \boxed{A_2 = 0}$$

KV

$$\Rightarrow x_p = -\frac{1}{3}\cos 2t$$

$$\Longrightarrow \quad x_{allg} = x_p + x_h \Rightarrow (6) \quad \boxed{x_{allg} = C_1 \sin t + C_2 \cos t - \frac{1}{3}\cos 2t}$$

$$(1) \Leftrightarrow (7): \quad y = -\dot{x} + \sin 2t$$

$$(6) \Rightarrow (8): \quad \dot{x} = C_1 \cos t - C_2 \sin t + \frac{2}{3}\sin 2t$$

$$(8) \text{ in } (7): \quad y = -C_1 \cos t + C_2 \sin t - \frac{2}{3}\sin 2t + \sin 2t$$

$$\Longrightarrow y = -C_1 \cos t + C_2 \sin t + \frac{1}{3}\sin 2t$$

$$\left.\begin{array}{l} x(0) = 1 \\ x(0) = C_2 - \frac{1}{3} \end{array}\right\} \Longrightarrow \boxed{C_2 = \frac{4}{3}}$$

$$\left.\begin{array}{l} y(0) = 0 \\ y(0) = -C_1 \end{array}\right\} \Longrightarrow C_1 = 0$$

$$\Longrightarrow \begin{pmatrix} x(t) \\ y(t) \end{pmatrix} = \begin{pmatrix} \frac{4}{3}\cos t - \frac{1}{3}\cos 2t \\ \frac{4}{3}\sin t + \frac{1}{3}\sin 2t \end{pmatrix}$$

$$= \frac{4}{3}\begin{pmatrix} \cos t \\ \sin t \end{pmatrix} + \frac{1}{3}\begin{pmatrix} -\cos 2t \\ \sin 2t \end{pmatrix}$$

$\boxed{A23}$

$\underline{\dot{x}} = \underline{A}\,\underline{x}$ mit $A = \begin{pmatrix} 1 & -1 \\ \alpha & -3 \end{pmatrix}$

$$|\underline{A} - \lambda \underline{E}| = 0 \Leftrightarrow \begin{vmatrix} 1-\lambda & -1 \\ \alpha & -3-\lambda \end{vmatrix} = 0$$

$$\Leftrightarrow \boxed{\lambda^2 + 2\lambda - 3 + \alpha = 0}$$

$$\lambda_{1/2} = -1 \pm \sqrt{4-\alpha}$$

λ reell $\Leftrightarrow 4 - \alpha \geq 0 \Leftrightarrow \boxed{\alpha \leq 4}$

λ imaginär $\Leftrightarrow \boxed{\alpha > 4}$

λ reell: ($\alpha \leq 4$)

$\boxed{\alpha = 4}$: $\lambda_1 = \lambda_2 = -1$

$\{e^{-t}, t \cdot e^{-t}\} \to 0 \quad (t \to \infty)$

(System stabil)

$\boxed{\alpha < 4}$ $\quad \lambda_{1/2} = -1 \pm \sqrt{4-\alpha}$

$\{e^{\lambda_1 t}, e^{\lambda_2 t}\}$

System stabil $\Leftrightarrow \lambda_{1/2} < 0$

$\Leftrightarrow -1 \pm \sqrt{4-\alpha} < 0 \Leftrightarrow \pm\sqrt{4-\alpha} < 1$

$\Leftrightarrow 4 - \alpha < 1 \Leftrightarrow \boxed{\alpha > 3}$

λ imaginär: ($\alpha > 4$)

$\lambda_{1/2} = -1 \pm j\sqrt{\alpha - 4}$

$\{e^{-t}\cos\sqrt{\alpha-4}\cdot t \,;\, e^{-t}\sin\sqrt{\alpha-4}\cdot t\}$

$\to 0 \quad (t \to \infty) \quad \forall \alpha \in \mathbb{R}$

$\Longrightarrow$ System stabil für $\boxed{\forall \alpha > 3}$

A24, S. 88

$$\ddot{x}_1 + 2ax_1 + ax_2 + b\dot{x}_1 = \cos\omega t$$

$$\ddot{x}_2 + 2ax_2 + ax_1 + b\dot{x}_2 = 0$$

$$\{z_1 = x_1,\ z_2 = \dot{x}_1,\ z_3 = x_2,\ z_4 = \dot{x}_2\}$$

$$x_1 = z_1$$

$$\dot{x}_1 = \dot{z}_1 = z_2$$

$$\ddot{x}_1 = \dot{z}_2 = -2az_1 - az_3 - bz_2 + \cos\omega t$$

$$\dot{x}_2 = \dot{z}_3 = z_4$$

$$\ddot{x}_2 = \dot{z}_4 = -2az_3 - az_1 - bz_4$$

$$\Rightarrow \boxed{\dot{\underline{z}} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -2a & -b & -a & 0 \\ 0 & 0 & 0 & 1 \\ -a & 0 & -2a & -b \end{pmatrix} \underline{z} + \begin{pmatrix} 0 \\ \cos\omega t \\ 0 \\ 0 \end{pmatrix}}$$

A25

(1) $\ddot{x} + 3x - 2\dot{y} = 0 \Rightarrow$ (3) $\dddot{x} + 3\dot{x} - 2\ddot{y} = 0$

(2) $\ddot{y} + 3y + 2\dot{x} = 0$ $\Rightarrow$ (4) $\ddddot{x} + 3\ddot{x} - 2\dddot{y} = 0$

(2) $\Rightarrow$ (5) $\dddot{y} + 3\dot{y} + 2\ddot{x} = 0 \Rightarrow$ (6) $\dddot{y} = -3\dot{y} - 2\ddot{x}$

(1) $\Leftrightarrow$ $2\dot{y} = \ddot{x} + 3x \Leftrightarrow$ (7) $\dot{y} = \frac{\ddot{x}}{2} + \frac{3}{2}x$

(6) in (4): $\ddddot{x} + 3\ddot{x} + 6\dot{y} + 4\ddot{x} = 0$

$\Leftrightarrow$ (8) $\ddddot{x} + 7\ddot{x} + 6\dot{y} = 0$

(7) in (8): $\ddddot{x} + 7\ddot{x} + 3\ddot{x} + 9x = 0$

$$\Leftrightarrow \boxed{\ddddot{x} + 10\ddot{x} + 9x = 0}$$

char. Pol.: $\lambda^4 + 10\lambda^2 + 9 = 0$

$w := \lambda^2 \Rightarrow w^2 + 10w + 9 = 0$

$\Rightarrow w_{1/2} = \frac{-10 \pm \overbrace{\sqrt{100-36}}^{8}}{2} \Rightarrow w_1 = -1,\ w_2 = -9$

$\Rightarrow$ R.S. $\lambda^2 = -1 \Rightarrow \lambda_1 = j,\ \lambda_2 = -j$

$\lambda^2 = -9 \Rightarrow \lambda_3 = 3j,\ \lambda_4 = -3j$

F.S. $\{\cos t, \sin t, \cos 3t, \sin 3t\}$

$\Rightarrow$ <u>allg. Lsg.</u>

$$(9)\quad x(t) = C_1 \cos t + C_2 \sin t + C_3 \sin 3t + C_4 \cos 3t$$

$$\Rightarrow \dot{x}(t) = -C_1 \sin t + C_2 \cos t + 3C_3 \cos 3t - 3C_4 \sin 3t$$

(10)

$$\Rightarrow (11)\quad \ddot{x}(t) = -C_1 \cos t - C_2 \sin t - 9C_3 \sin 3t - 9C_4 \cos 3t$$

(10), (11) in (7):
$$\dot{y} = \frac{\ddot{x}}{2} + \frac{3}{2}x$$

$$\Rightarrow \dot{y} = -\frac{C_1}{2}\cos t - \frac{C_2}{2}\sin t - \frac{9}{2}C_3 \sin 3t - \frac{9}{2}C_4 \cos 3t$$
$$+ \frac{3}{2}C_1 \cos t + \frac{3}{2}C_2 \sin t + \frac{3}{2}C_3 \sin 3t + \frac{3}{2}C_4 \cos 3t$$

$$= C_1 \cos t + C_2 \sin t - 3C_3 \sin 3t - 3C_4 \cos 3t$$

$$\Rightarrow y = C_1 \sin t - C_2 \cos t + C_3 \cos 3t - C_4 \sin 3t$$

$$\left.\begin{array}{l} x(0) = 1 \\ x(0) = C_1 + C_4 \end{array}\right\} \Rightarrow \boxed{C_1 + C_4 = 1}$$

$$\left.\begin{array}{l} \dot{x}(0) = 0 \\ \dot{x}(0) = C_2 + 3C_3 \end{array}\right\} \Rightarrow \boxed{C_2 + 3C_3 = 0}$$

$$\left.\begin{array}{l} y(0) = 0 \\ y(0) = -C_2 + C_3 \end{array}\right\} \Rightarrow \boxed{-C_2 + C_3 = 0}$$

$$\left.\begin{array}{l} \dot{y}(0) = 1 \\ \dot{y}(0) = C_1 - 3C_4 \end{array}\right\} \Rightarrow \boxed{C_1 - 3C_4 = 1}$$

$$\Rightarrow \boxed{C_4 = 0} \quad \boxed{C_1 = 1} \quad \boxed{C_3 = 0} \quad \boxed{C_2 = 0}$$

$\boxed{A26}$

$\lambda_1 = -1 + j$ ist EW $\Rightarrow$ $\lambda_2 = -1 - j$ ist EW

d. char. Polynoms von $|\underline{A} - \lambda \underline{E}| = 0$

<u>allg. Lsg.:</u>

$$\underline{\tilde{x}} = \begin{pmatrix} 1 \\ j \end{pmatrix} e^{-t} (\cos t + j \sin t)$$

$$= e^{-t} \begin{pmatrix} \cos t + j \sin t \\ -\sin t + j \cos t \end{pmatrix}$$

$$= \underbrace{e^{-t} \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix}}_{=: \underline{x}^{(1)}} + j \underbrace{e^{-t} \begin{pmatrix} \sin t \\ \cos t \end{pmatrix}}_{=: x^{(2)}}$$

$$\boxed{\begin{array}{l} \underline{x}^{(1)} = \mathrm{Re}(\underline{\tilde{x}}) \\ \underline{x}^{(2)} = \mathrm{Im}(\underline{\tilde{x}}) \end{array}}$$

$\Rightarrow$ allg. Lsg: $\boxed{\underline{x}(t) = K_1 x^{(1)} + K_2 x^{(2)}}$ $(K_1, K_2 \in \mathbb{R})$

$$\underline{\dot{x}}^{(1)} = \begin{pmatrix} -e^{-t} \cos t - e^{-t} \sin t \\ e^{-t} \sin t - e^{-t} \cos t \end{pmatrix} = e^{-t} \begin{pmatrix} \cos t - \sin t \\ \sin t - \cos t \end{pmatrix}$$

$x^{(1)}, \underline{\dot{x}}^{(1)}$ in $\underline{\dot{x}} = A \underline{x}$:

$$\cancel{e^{-t}} \begin{pmatrix} -\cos t - \sin t \\ \sin t - \cos t \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & -1 \end{pmatrix} \begin{pmatrix} \cos t \\ -\sin t \end{pmatrix} \cancel{e^{-t}}$$

$\boxed{a_{11} = -1}$

$$\Rightarrow \left.\begin{array}{l} -\cos t - \sin t = a_{11} \cos t - a_{12} \sin t \\ \sin t - \cos t = a_{21} \cos t + \sin t \end{array}\right\} \xRightarrow{\text{KV.}}$$

$\boxed{a_{12} = 1}$

$\boxed{a_{21} = -1}$

$$\boxed{\begin{aligned} x(t) &= \cos t \\ y(t) &= \sin t \end{aligned}}$$