$\boxed{A2}$ $z = f(x,y) = \frac{1}{1+x^2+y^2}$;

$B := \{(x,y) \in \mathbb{R}^2 \mid |x| \le 4 ; |y| \le 4\}$

<u>Höhenlinien</u>

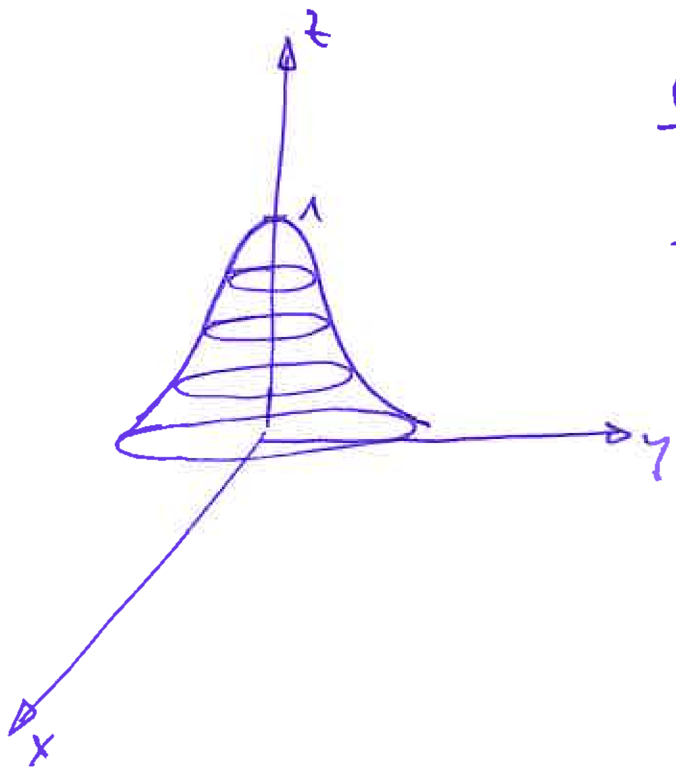
$z = \frac{1}{9} \Rightarrow \frac{1}{1+x^2+y^2} = \frac{1}{9} \Rightarrow x^2+y^2 = 8$

(Kreis um 0 mit $r = 2\sqrt{2}$)

$z = \frac{1}{3} \Rightarrow \frac{1}{1+x^2+y^2} = \frac{1}{3} \Rightarrow x^2+y^2 = 2$

(Kreis um 0 mit $r = \sqrt{2}$)

$z = 1 \Rightarrow \frac{1}{1+x^2+y^2} = 1 \Rightarrow x^2+y^2 = 0 \Rightarrow$ Ursprung $(0|0)$



<u>Glockenförmige Fläche</u>

$\boxed{A3}$ geg. $z = f(x,y) = \sqrt{9-x^2-y^2}$

$P(2|1|z_0)$ mit $z_0 = f(2,1) = 2$

$$\frac{\partial z}{\partial y} = \frac{-2y}{2\sqrt{9-x^2-y^2}} = -\frac{y}{\sqrt{9-x^2-y^2}}$$

$$\Rightarrow \tan\alpha = \left.\frac{\partial z}{\partial y}\right|_{(2,1)} = -\frac{1}{2} \Rightarrow \boxed{\alpha = -26{,}57^\circ}$$

$\boxed{A13}$ $z = \ln\sqrt{(x-a)^2+(y-b)^2}$ $\quad (a,b = \text{const.})$

$$\Rightarrow z = \frac{1}{2}\ln\left((x-a)^2+(y-b)^2\right)$$

z.z.: $z_{xx} + z_{yy} = 0$

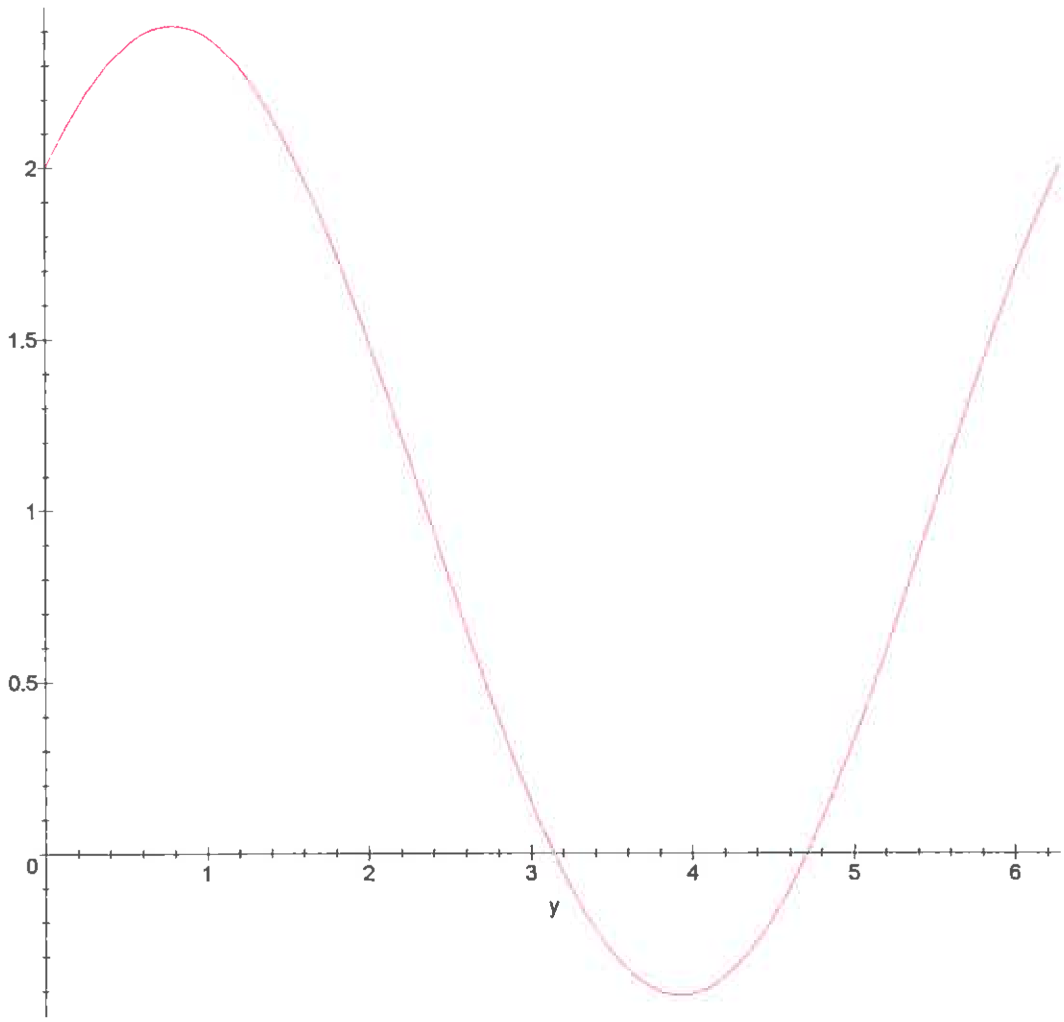
$$z_x = \frac{1}{2}\,\frac{1\cdot 2(x-a)}{(x-a)^2+(y-b)^2} = \frac{x-a}{(x-a)^2+(y-b)^2}$$

$$z_y = \frac{y-b}{(x-a)^2+(y-b)^2} \Rightarrow z_{xx} = \frac{(x-a)^2+(y-b)^2-(x-a)\cdot 2(x-a)}{\left[(x-a)^2+(y-b)^2\right]^2}$$

$$= \frac{(y-b)^2-(x-a)^2}{\left[(x-a)^2+(y-b)^2\right]^2}; \quad z_{yy} = \frac{(x-a)^2+(y-b)^2-(y-b)\cdot 2(y-b)}{\left[(x-a)^2+(y-b)^2\right]^2} =$$

$$= \frac{(x-a)^2-(y-b)^2}{(x-a)^2+(y-b)^2} \Rightarrow z_{xx}+z_{yy} = 0 \quad \square$$

```
> plot(1+sin(y)+sin(Pi/2 + y),y=0..2*Pi);
```




```
>
```

$\boxed{A4}$ a)

$\boxed{A4}$

b) <u>geg.</u>: $z = f(x,y) = \sin x + \sin y + \sin(x+y)$

$P\left(\frac{\pi}{2} \middle| \frac{\pi}{4} \middle| z_0\right)$ mit $z_0 = f\left(\frac{\pi}{2}, \frac{\pi}{4}\right) = 1 + \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = 1 + \sqrt{2}$

$$\frac{\partial z}{\partial y} = \cos y + \cos(x+y)$$

$$\left.\frac{\partial z}{\partial y}\right|_{\left(\frac{\pi}{2}, \frac{\pi}{4}\right)} = \cos\left(\frac{\pi}{4}\right) + \cos\left(\frac{3}{4}\pi\right) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} = 0$$

$\Rightarrow \tan\alpha = 0 \Rightarrow \boxed{\alpha = 0^\circ}$

c) $\frac{\partial f}{\partial x} = \cos x + \cos(x+y)$

$\frac{\partial f}{\partial y} = \cos y + \cos(x+y)$

Es gilt: $\left.\frac{\partial f}{\partial x}\right|_{\left(\frac{\pi}{3}, \frac{\pi}{3}\right)} = \cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{2}{3}\pi\right) = 0{,}5 - 0{,}5 = 0$

$$\left.\frac{\partial f}{\partial y}\right|_{\left(\frac{\pi}{3}, \frac{\pi}{3}\right)} = \cos\left(\frac{\pi}{3}\right) + \cos\left(\frac{2}{3}\pi\right) = 0$$

$\Rightarrow$ A. d. Stelle $x_0 = y_0 = \frac{\pi}{3}$ liegt eine waagr. Tang. vor.

$\boxed{A6}$ $u = u(x,y,z) = xy + yz + zx$

a) $x = 3;\ y = 3;\ z = 1;\ dx = 0{,}1;\ dy = -0{,}2;\ dz = 0{,}2$

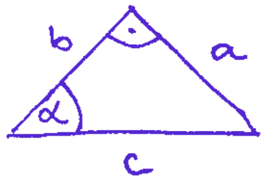
$$\Rightarrow \frac{\partial u}{\partial x} = y + z\ ;\quad \frac{\partial u}{\partial y} = x + z\ ;\quad \frac{\partial u}{\partial z} = y + x$$

$$\Delta u = u(2.1;\ 2.8;\ 1.2) - u(2,3,1) = \underline{\underline{0{,}76}}$$

b) $$du = \frac{\partial u}{\partial x}dx + \frac{\partial u}{\partial y}dy + \frac{\partial u}{\partial z}dz$$

$$\Rightarrow du = \left.\frac{\partial u}{\partial x}\right|_{(2,3,1)} dx + \left.\frac{\partial u}{\partial y}\right|_{(2,3,1)} dy + \left.\frac{\partial u}{\partial z}\right|_{(2,3,1)} dz = \underline{\underline{0{,}8}}$$

$\boxed{A8}$



$b = 28\,m \qquad \Delta b = 0{,}05\,m$

$c = 35\,m \qquad \Delta c = -0{,}1\,m$

$$\cos\alpha = \frac{b}{c} \Rightarrow \alpha = \arccos\left(\frac{b}{c}\right) \Rightarrow \alpha = \alpha(b,c)$$

$$\frac{\partial\alpha}{\partial b} = -\frac{1}{\sqrt{1-\frac{b^2}{c^2}}}\cdot\frac{1}{c} = -\frac{1}{\sqrt{c^2-b^2}}$$

$$\frac{\partial\alpha}{\partial c} = -\frac{1}{\sqrt{1-\frac{b^2}{c^2}}}\left(-\frac{b}{c^2}\right) = \frac{b}{c\sqrt{c^2-b^2}}$$

$$\Rightarrow d\alpha = \frac{\partial\alpha}{\partial b}\Delta b + \frac{\partial\alpha}{\partial c}\Delta c \text{ ; Es gilt: } \Delta\alpha \approx d\alpha$$

$$\Rightarrow d\alpha = -\frac{1}{\sqrt{c^2-b^2}}\Delta b + \frac{b}{c\sqrt{c^2-b^2}}\Delta c \approx -0{,}0062 \mathrel{\hat{=}} \underline{\underline{-0{,}35^\circ}}$$

$\boxed{A11}$ $z = x^3 + axy^2 \text{ , } a \in \mathbb{R}$

$$\Rightarrow z_x = 3x^2 + ay^2 \Rightarrow z_{xx} = 6x$$

$$z_y = 2axy \Rightarrow z_{yy} = 2ax$$

$$3x - ax = 0$$

$$z_{xx} + z_{yy} = 0 \Rightarrow \underset{6x}{\cancel{2axy}} + 2ax = 0 \Rightarrow \cancel{a(xy+x) = 0}$$

$$\Rightarrow x(3-a) = 0 \Rightarrow x = 0 \lor \boxed{a = 3}$$

$\boxed{A\,9}$ $G = G(\gamma, D, r, \phi)$

a) $\Rightarrow G = \frac{1}{2}\gamma D r^2 (\phi - \sin\phi)$

$$\Rightarrow \frac{\partial G}{\partial r} = \gamma D r(\phi - \sin\phi) \; ; \; \frac{\partial G}{\partial \phi} = \frac{1}{2}\gamma D r^2 (1 - \cos\phi)$$

$$\frac{\partial G}{\partial \gamma} = \frac{1}{2} D r^2 (\phi - \sin\phi) \; ; \; \frac{\partial G}{\partial D} = \frac{1}{2}\gamma r^2 (\phi - \sin\phi)$$

$$\Delta G \approx dG = \frac{\partial G}{\partial r}\Delta r + \frac{\partial G}{\partial \gamma}\Delta\gamma + \frac{\partial G}{\partial D}\Delta D + \frac{\partial G}{\partial \phi}\Delta\phi$$

$$\left|\frac{\Delta G_{max}}{G}\right| = \left|\frac{2}{r}\Delta r\right| + \left|\frac{1}{\gamma}\Delta\gamma\right| + \left|\frac{1}{D}\Delta D\right| + \left|\frac{1-\cos\phi}{\phi - \sin\phi}\Delta\phi\right|$$

<u>(max. rel. Fehler)</u>

$\gamma = 8{,}75 \frac{g}{cm^3}$; $\Delta\gamma = 0{,}02 \frac{g}{cm^3}$

$r = 10$ cm ; $\Delta r = 0{,}01$ cm

$D = 0{,}1$ cm ; $\Delta D = 0{,}005$ cm

$\phi = 60°$; $\Delta\phi = 0{,}05°$

$\phi = 60° \mathrel{\hat{=}} 1{,}0472$; $\Delta\phi = 0{,}05° \mathrel{\hat{=}} 0{,}0009$

$$\Rightarrow \left|\frac{\Delta G_{max}}{G}\right| = 0{,}0568 \mathrel{\hat{=}} 5{,}7\,\%$$

b) Die Meßg. muß vor allem bei D gesteigert werden, da dieser <u>rel. Fehler</u> <u>groß</u> ist

$\boxed{A10}$

$$S = \frac{3r\sin(\alpha)}{3\alpha} = s(r,\alpha)$$

$$S = \frac{2 \cdot 10 \sin(0{,}7854)}{3 \cdot 0{,}7854} = 6{,}0021 \text{ cm}$$

$$6{,}0021 = \frac{20{,}2 \sin(0{,}7854 + \Delta\alpha)}{3(0{,}7854 + \Delta\alpha)}$$

$$\Rightarrow 20{,}2 \sin(0{,}7854 + \Delta\alpha) = 18{,}0063(0{,}7854 + \Delta\alpha)$$

$$\Rightarrow \sin(\underbrace{0{,}7854 + \Delta\alpha}_{=:x}) = 0{,}8514(0{,}7854 + \Delta\alpha)$$

$$\Rightarrow \sin x = 0{,}8514x \xrightarrow[\text{Newton-verfahren}]{} x \approx 0{,}82$$

$$\Rightarrow \Delta\alpha = \underbrace{0{,}82}_{x} - 0{,}7854 \mathrel{\hat{=}} 2{,}1^\circ$$

$$\Rightarrow \boxed{\alpha \simeq 47{,}1^\circ}$$